Propagation of uncertainty in the mechanical and biological response of growing tissues using multi-fidelity Gaussian process regression

Computer Methods in Applied Mechanics and Engineering

Supplementary materials

In the manuscript, we create surrogate models based on multi-fidelity Gaussian process regression independently for three different volumes, $V = 40$, 50, and 60 ml. All surrogates are validated first before we explore uncertainty propagation over the input space. In the main text we restrict our attention to 50 ml. However, for the other two volumes, 40 and 60 ml, we follow the same procedure to build, validate, and test the surrogate. The results for 40 and 60 ml are shown here. First, the effect of changing the number of high fidelity function evaluations is represented in Figure 1 and 2 for 40 and 60 ml, respectively. After the validation process, uncertainty propagation over the input space is carried out and shown in Figures 3 and 4 for the two volumes.

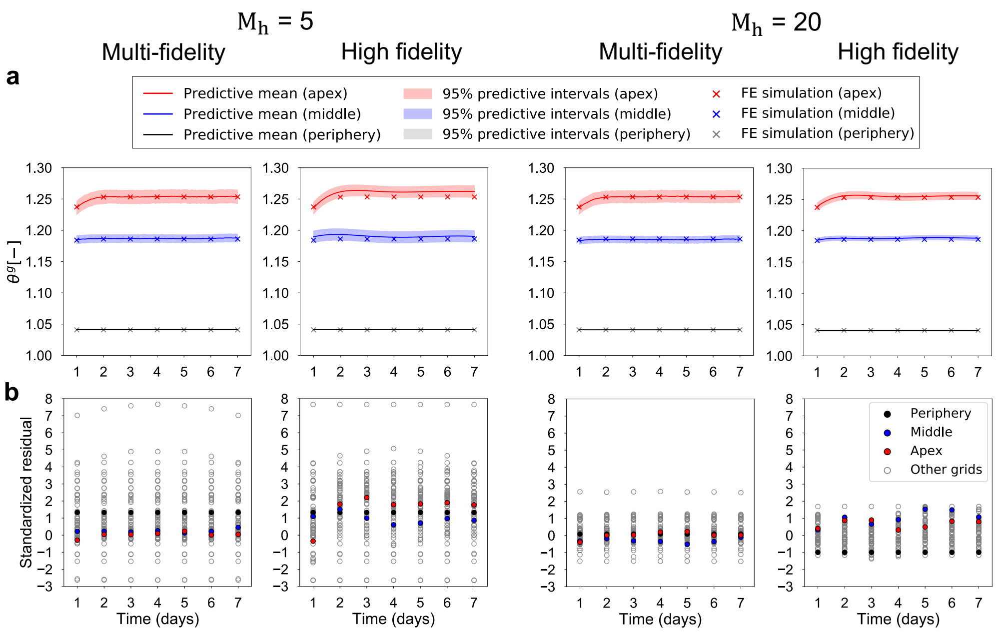

Figure 1: Effect of changing the number of high fidelity function evaluations on the performance of the multi-fidelity and high fidelity GPs when inflation volume $V = 40$ ml. Results for $M_h = 5$ high fidelity function evaluations are depicted in the first two columns, while the last two columns correspond to $M_h = 20$ high fidelity function evaluations during training. **a**) Prediction of the temporal evolution of the growth variable ϑ^g for three points of interest: apex, middle, and periphery of the expander. Plots show the predictive mean and confidence intervals alongside the truth (Finite element simulation based on the high fidelity model). **b**) Standardized residuals for all 100 spatial locations scattered with respect to time.

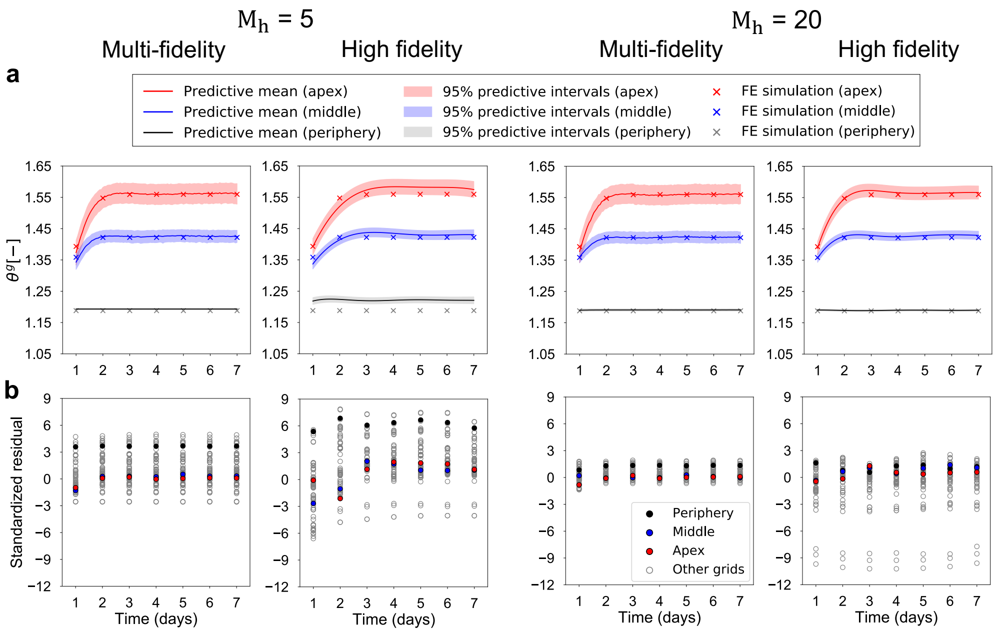

Figure 2: Effect of changing the number of high fidelity function evaluations on the performance of the multi-fidelity and high fidelity GPs when inflation volume $V = 60$ ml. Results for $M_h = 5$ high fidelity function evaluations are depicted in the first two columns, while the last two columns correspond to $M_h = 20$ high fidelity function evaluations during training. **a**) Prediction of the temporal evolution of the growth variable ϑ^g for three points of interest: apex, middle, and periphery of the expander. Plots show the predictive mean and confidence intervals alongside the truth (Finite element simulation based on the high fidelity model). **b**) Standardized residuals for all 100 spatial locations scattered with respect to time.

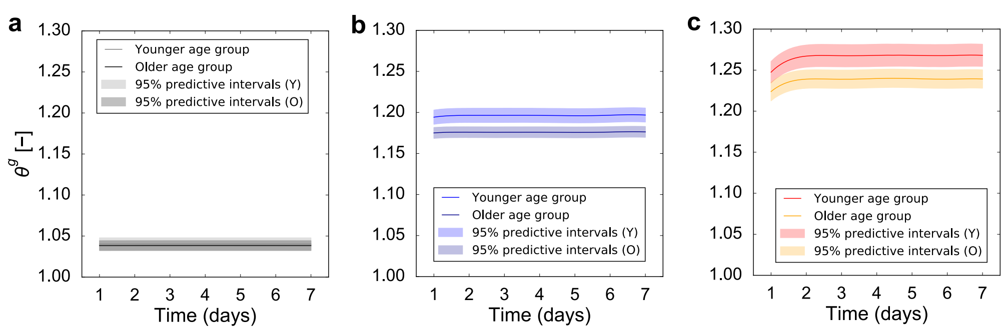

Figure 3: Propagation of mechanical response uncertainty on the resulting tissue growth at apex (**a**), middle (**b**), and periphery (**c**) points for the volume, $V = 40$ ml. We consider two age groups with different tissue parameter distribution of shear modulus, μ. For the younger group, the tissue parameter is normally distributed with mean $\mu_y = 0.3$ and standard deviation 0.051. For the older group, the mean is $\mu_o = 0.75$ and the standard deviation is 0.026. We sample these distributions and use the surrogate to predict the resulting growth at three locations of interest. The younger group shows greater growth in the prediction compared to the older group at middle (**b**) and apex (**c**) point, but periphery (**a**) point does not have large difference between groups. The overall growth is small and reached equilibrium within the first two days after inflation due to the small volume of the expander.

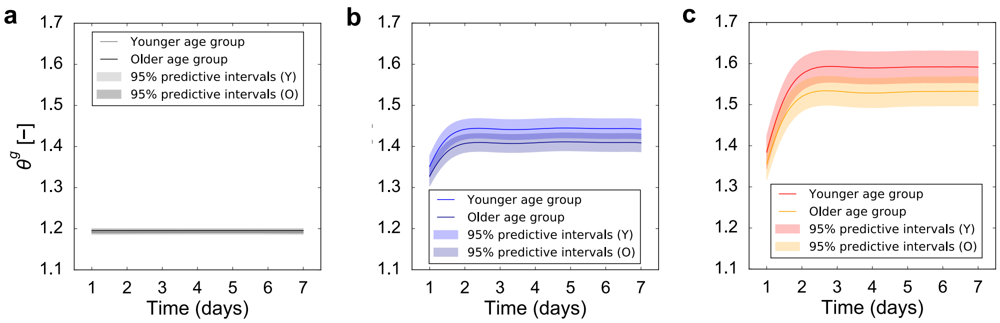

Figure 4: Propagation of mechanical response uncertainty on the resulting tissue growth at apex (**a**), middle (**b**), and periphery (**c**) points for the volume, $V = 60$ ml. We consider two age groups with different tissue parameter distribution of shear modulus, μ. For the younger group, the tissue parameter is normally distributed with mean $\mu_y = 0.3$ and standard deviation 0.051. For the older group, the mean is $\mu_o = 0.75$ and the standard deviation is 0.026. We sample these distributions and use the surrogate to predict the resulting growth at three locations of interest. The younger group shows greater growth in the prediction compared to the older group at middle (**b**) and apex (**c**) point, but periphery (**a**) point does not have large difference between groups. The adaptation for the middle point takes place within the first two or three days, while at the apex the response is slightly longer. The variance increases as we move from the periphery to the apex of the expander.